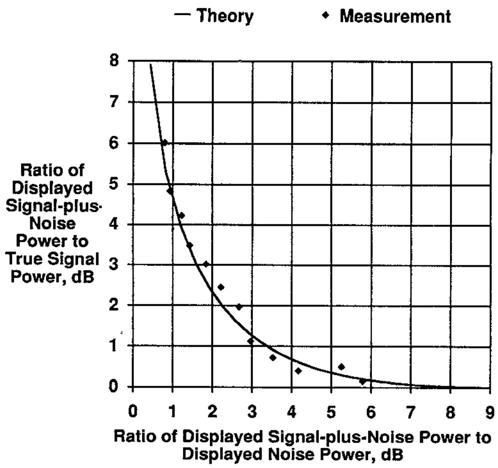

Fig. 4. Effect of noise on the displayed signal power when a spectrum analyzer is in logarithmic display mode.

Similarly, the ratio for the log mode corresponding to R_{sns} is

$$\rho_{\mathrm{sns}} \equiv \frac{\text{(displayed signal plus noise voltage)}}{\text{(displayed signal voltage if noise is absent)}}$$

$$= \frac{G}{2}\Bigg(- \ln m - \gamma + \exp(-m) \cdot \sum_{k=1}^{\infty} \frac{m^k}{k!}\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{k}\right)\Bigg). \tag{14}$$

A plot of ρ_{sns} versus ρ_{snn} is given in Fig. 4, along with measured data obtained on a Hewlett-Packard 8566B spectrum analyzer in the setup depicted in Fig. 2. As we did with Fig. 3 for the linear mode, we can use Fig. 4 for the logarithmic mode, as follows. First, find the measured ratio of displayed signal plus noise to displayed noise on the horizontal axis; the corresponding value on the vertical axis is the correction factor, which, when subtracted from the displayed signal-plus-noise power, yields the true signal power. For example, say one measures a noise power of -63 dBm and a signal-plus-noise power of -60 dBm for a ratio of signal plus noise to noise of 3 dB. Fig. 4 shows that the corresponding ratio of displayed signal-plus-noise power to true signal power is approximately 1.25 dB; hence the true signal voltage is 1.25 dB below the measured signal-plus-noise voltage, or -61.25 dBm.

Conclusion

A spectrum analyzer does not measure true power, but rather only the envelope of the voltage in the linear mode, or the log of the envelope of the voltage in the log mode. The corrections to be applied to arrive at the true signal power when noise is present are derived above. The corrections are shown graphically in Fig. 3 for the linear mode, and in Fig. 4 for the log mode. To use these graphs one finds the ratio of displayed signal plus noise to displayed noise on the horizontal scale; then the corresponding value on the vertical scale is the number of dB one must subtract from the displayed signal plus noise to arrive at the true signal level.

Acknowledgment

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Synthesis of Schiffman Phase Shifters

José Luis Ramos Quirarte and J. Piotr Starski

***Abstract*—The Schiffman phase shifter is a very useful passive component. In this paper equations to determine its phase deviation and maximum bandwidth when the coupling coefficient is known are presented. Equations are given also to determine the coupling coefficient for a desired bandwidth or maximum phase deviation.**

***Keywords*—Phase shifters; Schiffman sections; coupled transmission lines.**

I. Introduction

The Schiffman phase shifter [1]–[5] is a broadband differential phase shifter where the phase shift $\Delta\phi$, is obtained as a subtraction of the phase response of a coupled section with adjacent ports interconnected [6] and the phase response of a uniform line. i.e.:

$$\Delta\phi = K\theta - \cos^{-1}\left(\frac{\rho - \tan^2\theta}{\rho + \tan^2\theta}\right) \tag{1}$$

where θ is the electrical length of the coupled section and ρ is its *impedance ratio* defined as

$$\rho = \frac{Z_{0e}}{Z_{0o}}. \tag{2}$$

Z_{0e} and Z_{0o} are the even and odd mode impedances of the coupled section respectively. The input impedance Z_I, of the coupled section is matched at all frequencies and is given by

$$Z_I = \sqrt{Z_{0e} Z_{0o}}. \tag{3}$$

The *coupling* C, and the impedance ratio are related by [2], [7]:

$$C = -20 \log \frac{\rho - 1}{\rho + 1} \tag{4}$$

or

$$\rho = \frac{1 + 10^{-C/20}}{1 - 10^{-C/20}}. \tag{5}$$

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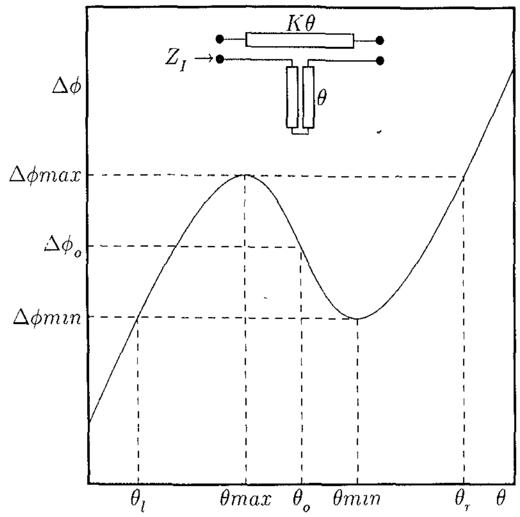

Fig. 1. Standard Schiffman phase shifter and its typical phase response.

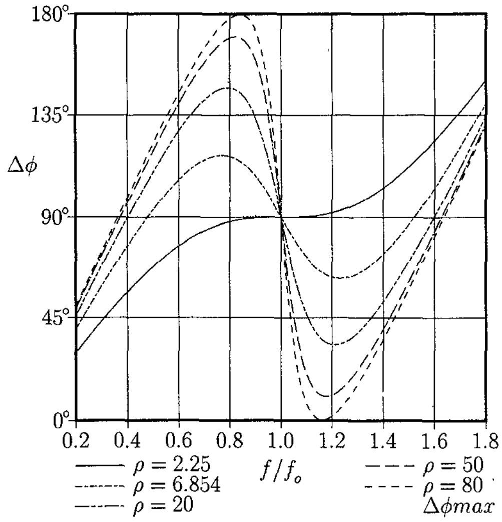

Fig. 2. Response of 90° Schiffman phase shifters for different coupling coefficients and the behavior of $\Delta\phi_{\max}$.

In order to get a desired phase performance, the constant K, θ, and ρ in (1) must be chosen properly. The problem in designing Schiffman phase shifters in this way, is that often several simulations or optimization are required.

II. Direct Design Formulas for Schiffman Phase Shifter

The Schiffman phase shifter (named Type-A Network in [1]) is shown in Fig. 1 as well as its typical phase response (given by (1)).

A direct method to design the Schiffman phase shifter is presented as follows. The first and second derivatives of the differential phase shift function in (1) with respect to θ are given by

$$\Delta\phi' = \frac{\partial(\Delta\phi)}{\partial\theta} = K - \frac{2\sqrt{\rho}\,(1+\tan^2\theta)}{\rho+\tan^2\theta} \tag{6}$$

and

$$\Delta\phi'' = \frac{\partial^2(\Delta\phi)}{\partial\theta^2} = \frac{4\sqrt{\rho}\,(\rho-1)\tan\theta(1+\tan^2\theta)}{(\rho+\tan^2\theta)^2}. \tag{7}$$

It can be seen from (7) that $\Delta\phi''=0$ when $\theta=90°$ and since $\tan\theta=-\tan(180°-\theta)$, the function $\Delta\phi$ is symmetric around $\theta=90°$ [8]. Thus the maximum bandwidth is obtained when $\theta=\theta_o=90°$ at the center frequency. Since $\rho>1$ we can also see from (7) that $\Delta\phi$ is convex for $\theta<90°$ and concave for $\theta>90°$.

From (6) we observe that in order to have a positive phase shift ($\Delta\phi>0°$), it is necessary that $K>2$.

The maximum (local maximum) of the function $\Delta\phi$ is obtained when $\Delta\phi'=0$ and using (6) the maximum will be located at

$$\theta_{\max} = \tan^{-1}\sqrt{\frac{K\rho-2\sqrt{\rho}}{2\sqrt{\rho}-K}} \tag{8}$$

and the respective minimum of the function is located at $\theta_{\min} = 180° - \theta_{\max}$.

From (8) we can see that $\Delta\phi$ has a maximum (where $\theta_{\max}$ is real) if

$$2\sqrt{\rho} > K \tag{9}$$

when $2\sqrt{\rho}=K$, we obtain the maximally flat-case, and for $2\sqrt{\rho}<K$ the function does not have a maximum.

If we substitute (8) in (1) we obtain an expression for the maximum of the function as

$$\Delta\phi_{\max} = K\theta_{\max} - \cos^{-1}\left(\frac{\rho-\tan^2\theta_{\max}}{\rho+\tan^2\theta_{\max}}\right) \tag{10}$$

Now we can define the phase deviation ϵ, as

$$\epsilon = \Delta\phi_{\max} - \Delta\phi_o = \Delta\phi_o - \Delta\phi_{\min} \tag{11}$$

where $\Delta\phi_o$ is the desired phase shift and $\Delta\phi_{\min}$ is the function $\Delta\phi$ evaluated at the frequency f_l ($f_l \equiv \theta_l$) for which the absolute value of the phase deviation is the same as that for $\Delta\phi_{\max}$. Thus using (1) and (11) we can write

$$\Delta\phi_{\min} = \Delta\phi_o - \epsilon = K\theta_l - \cos^{-1}\left(\frac{\rho-\tan^2\theta_l}{\rho+\tan^2\theta_l}\right) \tag{12}$$

and by symmetry the maximum frequency f_r ($f_r \equiv \theta_r$), at which the function has the same phase deviation is

$$\theta_r = 180° - \theta_l \tag{13}$$

and so the maximum bandwidth B, can be defined as

$$B = \frac{\theta_r-\theta_l}{\theta_o} = \frac{180°-2\theta_l}{90°}. \tag{14}$$

An interesting characteristic of $\Delta\phi_{\max}$ can be observed if we take the first derivative of $\theta_{\max}$ (8) with respect to ρ:

$$\theta'_{\max} = \frac{d(\theta_{\max})}{d\rho} = \frac{\rho+1-K\sqrt{\rho}}{2(\rho-1)\sqrt{\rho(K\rho-2\sqrt{\rho})(2\sqrt{\rho}-K)}}. \tag{15}$$

The derivative is equal to zero if

$$\rho = \frac{K^2-2+K\sqrt{K^2-4}}{2} \tag{16}$$

where the plus sign before the radical is the only possible choice since ρ must also satisfy the condition in (9).

Thus we observe that $\Delta\phi_{\max}$ moves from $\theta=90°$ (when $\rho=K^2/4$) to $\theta=\min\{\theta_{\max}\}$ (when ρ satisfies (16) and then returns to $\theta=90°$ for values of ρ greater than (16). In Fig. 2 the response of 90° Schiffman phase shifters has been plotted for different values of ρ. In that figure $\Delta\phi_{max}$ is also shown.

If it is desired to have the maximum of the phase function at some fixed frequency, we can determine the coupling by solving (8) for ρ:

$$\rho = \frac{2(1+\psi_m)^2}{K^2} - \psi_m \pm \sqrt{\left(-\frac{2(1+\psi_m)^2}{K^2} + \psi_m\right)^2 - \psi_m^2} \quad (17)$$

where

$$\psi_m = \tan^2\theta_{max}. \quad (18)$$

Equations (8), (17), and (4) give the relation between the impedance ratio ρ, or coupling C, and the location of the maximum of the phase function θ_{max}.

An expression for $\Delta\phi_{max}$ only as a function of ρ can be obtained if we substitute (8) in (10):

$$\Delta\phi_{max} = K\tan^{-1}\sqrt{\frac{K\rho - 2\sqrt{\rho}}{2\sqrt{\rho} - K}} - \cos^{-1}\left(\frac{\rho + 1 - K\sqrt{\rho}}{\rho - 1}\right). \quad (19)$$

Also, because $\Delta\phi'_{max}$ and $\Delta\phi_{min}$ are related by (11), we can obtain an expression relating θ_l and ρ by substituting (12) and (19) into (11):

$$\cos^{-l}\left(\frac{\rho - \psi_l}{\rho + \psi_l}\right) - K\theta_l = K\tan^{-1}\sqrt{\frac{K\rho - 2\sqrt{\rho}}{2\sqrt{\rho} - K}} - \cos^{-1}\left(\frac{\rho + 1 - K\sqrt{\rho}}{\rho - 1}\right) - 2\Delta\phi_o \quad (20)$$

where

$$\psi_l = \tan^2\theta_l. \quad (21)$$

The presented equations could be used as follows: when the coupling coefficient is known, the phase response is obtained by (1) or (8)–(14) can be used to calculate the bandwidth and phase deviation. When the coupling coefficient is unknown, the following procedure is suggested.

III. Procedure for Synthesis of Coupling Coefficient

When a special value for the phase deviation or maximum bandwidth is desired, the coupling required to satisfy that condition is found as follows.

A. Procedure When the Phase Deviation is Chosen

1) When the phase deviation has been fixed, the value of K can be calculated for the desired shift at the center frequency by using (1).
2) From (11) $\Delta\phi_{max}$ and $\Delta\phi_{min}$ can be determined.
3) ρ is obtained by using (19) and,
4) Using (12) θ_l is calculated.
5) And by (14) the maximum bandwidth is obtained.

B. Procedure When the Bandwidth is Chosen

1) K is determined from (1) for the desired phase shift at the center frequency.
2) From (14) θ_l is determined.
3) The impedance ratio and the coupling in dB's are calculated from (20)–(21) and (4).
4) The phase deviation is obtained by (12).

C. Procedure When Weak Coupling is Desired

If it is desired to have weak coupling ($\rho \leqslant K^2/4$), both the phase deviation and maximum bandwidth must be specified, and from those values the coupling is determined. To select these parameters we do as follows:

D. The Phase Deviation is Chosen as Desired

1) K is determined from (1) for the desired phase shift at the center frequency.
2) Using (12) with $\rho = K^2/4$, the minimum allowed value for θ_l is obtained.
3) Thus any value for θ_l greater or equal that minimum, could be used in (12) along the selected phase deviation to find the actual value for ρ.
4) And the bandwidth is given by (14).

E. The Maximum Bandwidth is Chosen as Desired

1) K is determined from (1) for the desired phase shift at the center frequency.
2) Using (14) we obtain the value for θ_l.
3) Using θ_l and $\rho = K^2/4$ in (12), the minimum allowed value for the phase deviation is found.
4) Any phase deviation greater or equal that minimum could be used along θ_l into (12) to find the actual value for ρ, and C in dB's is obtained from (4).

Examples: Design 90° Schiffman phase shifters having: 1) $\epsilon = 2°$, (find the maximum bandwidth and give the coupling that satisfies that condition) and 2) $B = 30\%$, (give the maximum phase deviation and obtain the required coupling).

Here we follow the procedure III-A Using (1) with $\theta_o' = \Delta\phi_o = 90°$ we get:

$$K = 3.$$

From (11) we find

$$\Delta\phi_{max} = 92°,$$

and

$$\Delta\phi_{min} = 88°.$$

From (19) it is found that

$$\rho \approx 2.625 \quad (C \approx 6.97 \text{ dB})$$

using (12) we get

$$\theta_l \approx 63°,$$

and finally the bandwidth is obtained from (14) as

$$B = 0.6 = 60\%.$$

The procedure III-B is applied in this case. From (14) we find

$$\theta_l = 76.5°.$$

From (21),

$$\psi_l = 17.3497$$

using these values in (20) we get

$$\rho \approx 2.333 \quad (C \approx 7.96 \text{ dB}),$$

and from (12) the phase deviation is obtained as

$$\epsilon = 0.224°.$$

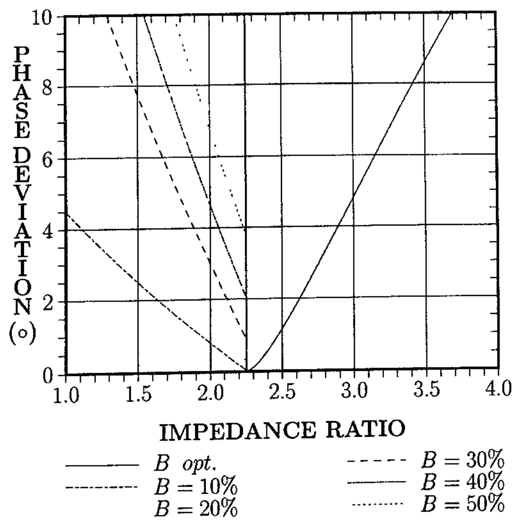

Fig. 3. Phase deviation versus impedance ratio for the 90° Schiffman phase shifter.

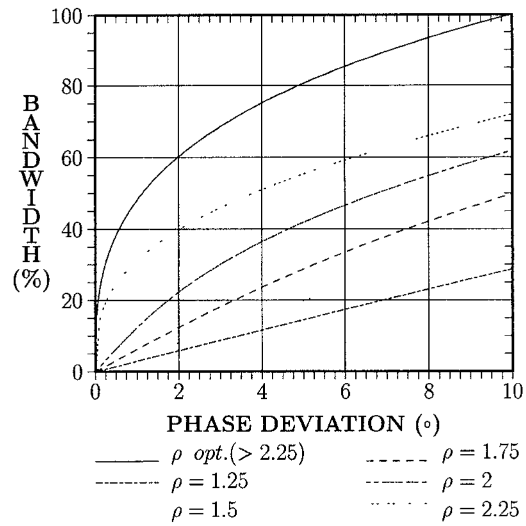

Fig. 5. Bandwidth versus phase deviation for the 90° Schiffman phase shifter.

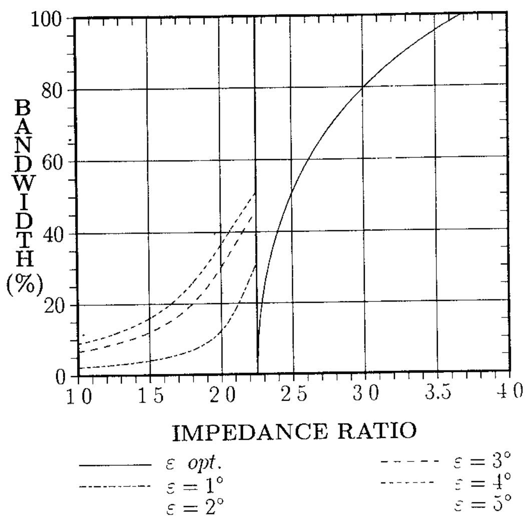

Fig. 4. Bandwidth versus impedance ratio for the 90° Schiffman phase shifter.

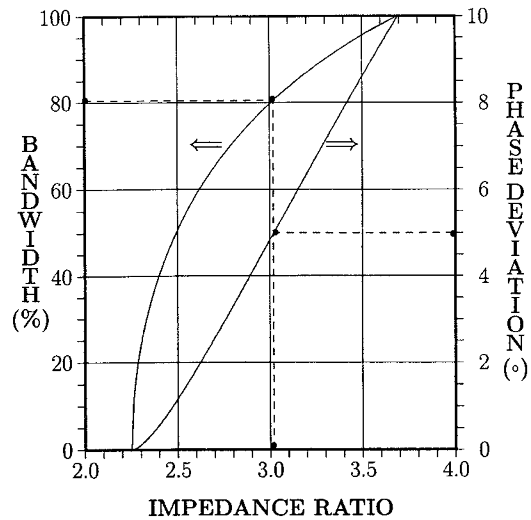

Fig. 6. Optimum relationship between bandwidth, phase deviation and impedance ratio for the 90° Schiffman phase shifter.

IV. Design Curves

Since the 90° phase shifter is a very common case, in order to simplify its design, we use the equations and procedures presented to obtain figures that can be used directly in the design.

In Fig. 3 the relation between the phase deviation in degrees and the impedance ratio is shown for the optimum case B_{opt} (when $\rho > 2.25$) as well as other cases having a fixed bandwidth (when $\rho < 2.25$).

In Fig. 4 the relation between the percentage bandwidth and the impedance ratio is shown for the optimum case ϵ_{opt} (when $\rho > 2.25$) as well as other cases having a fixed phase deviation (when $\rho < 2.25$).

In Fig. 5 the relation between the percentage bandwidth and the phase deviation in degrees is shown for the optimum case ρ_{opt} (when $\rho > 2.25$) as well as other cases having a fixed impedance ratio (when $\rho < 2.25$).

And finally the optimum relation (when $\rho > 2.25$) between the percentage bandwidth, the impedance ratio and the phase deviation in degrees is shown in Fig. 6.

V. Conclusion

Formulas for synthesis of Schiffman phase shifters have been presented. The formulas make possible the calculation of the bandwidth and maximum phase deviation for a given coupling coefficient or the calculation of the coupling of the coupled section in order to obtain specified phase deviation or maximum bandwidth.

Design curves for 90° Schiffman phase shifter are also presented.

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A Microstrip Line on a Chiral Substrate

Michael S. Kluskens and Edward H. Newman

Abstract—**Right and left circular vector potentials are developed and used in a spectral-domain solution for a microstrip transmission line on a chiral substrate. These vector potentials have properties similar to those of the usual magnetic and electric vector potentials, except that they result in circular rather than linearly polarized fields, thereby simplifying field expansions in chiral media. The chiral microstrip line does not have bifurcated modes like other chiral guided wave structures; however, the chiral substrate causes a significant asymmetry in both the fields and currents.**

I. Introduction

This paper presents a spectral-domain Galerkin moment method (MM) solution for a microstrip transmission line on a chiral substrate. A chiral medium is a form of artificial dielectric consisting of chiral objects randomly embedded in a dielectric or other medium [1]. At optical frequencies, the chiral objects are molecules and the medium is called an isotropic optically active medium. At microwave frequencies, early research used conducting helices as a scale model for optical activity [2]. From this and later work, the constitutive relationships for chiral media have been shown to be the same as those for isotropic optically active media; therefore, the same notation is used [3, sec. 8.3].

A chiral medium is distinguished from other media in that right and left circularly polarized waves propagate through it with different phase velocities, even though it is a reciprocal and isotropic medium. For most chiral guided wave structures this property results in bifurcated modes [4]–[6], i.e., pairs of modes with the same cutoff frequency. The chiral microstrip line does not have bifurcated modes, and thus the dispersion curves are single valued. The primary effect of the chiral substrate is to generate asymmetric longitudinal and symmetric transverse fields. This effect could significantly alter the properties of microwave devices constructed on a chiral substrate.

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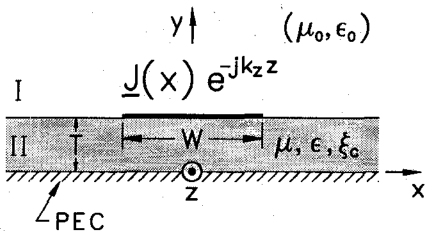

Fig. 1. Microstrip line on a grounded chiral substrate.

II. Theory

The constitutive relationships for a chiral medium can be written as

$$\boldsymbol{D} = \epsilon_c \boldsymbol{E} - j\mu\xi_c \boldsymbol{H} \tag{1}$$

$$\boldsymbol{B} = \mu \boldsymbol{H} + j\mu\xi_c \boldsymbol{E} \tag{2}$$

where $\epsilon_c = \epsilon + \mu\xi_c^2$, μ is the permeability, ϵ is the permittivity, and the pseudoscalar ξ_c is the chirality admittance of the medium ($e^{j\omega t}$).

Following the techniques used in [7], [8], the right ($\boldsymbol{R}$) and left ($\boldsymbol{L}$) circular vector potentials are defined as

$$\boldsymbol{R} = \hat{a}\psi(k_R) \tag{3}$$

$$\boldsymbol{L} = \hat{a}\psi(k_L) \tag{4}$$

where $\hat{a}$ is an arbitrary unit vector and $\psi(k)$ is a solution of the scalar wave equation $\nabla^2\psi(k) + k^2\psi(k) = 0$. The right and left circularly polarized electric fields are formed using

$$\boldsymbol{E}_R = \nabla \times \left(\boldsymbol{R} + \frac{1}{k_R} \nabla \times \boldsymbol{R} \right) \tag{5}$$

$$\boldsymbol{E}_L = \nabla \times \left(\boldsymbol{L} - \frac{1}{k_L} \nabla \times \boldsymbol{L} \right) \tag{6}$$

where the wave numbers k_R and k_L are given by

$$\left. \begin{matrix} k_R \\ k_L \end{matrix} \right\} = \omega\sqrt{\mu\epsilon_c} \pm \omega\mu\xi_c. \tag{7}$$

The corresponding magnetic fields are given by

$$\begin{pmatrix} \boldsymbol{H}_R \\ \boldsymbol{H}_L \end{pmatrix} = \frac{j}{\eta_c} \begin{pmatrix} \boldsymbol{E}_R \\ -\boldsymbol{E}_L \end{pmatrix} \tag{8}$$

where $\eta_c = \sqrt{\mu/\epsilon_c}$ is the chiral wave impedance. The right (or left) circular vector potential component R_y (or L_y) produces a right (or left) circular to y field RC_Y (or LC_Y), just as the magnetic vector potential component A_y produces a transverse magnetic to y field TM_Y.

The microstrip line is shown in Fig. 1, where the substrate has parameters (μ, ϵ, ξ_c) and thickness T. The microstrip line is W wide, infinitely thin, and perfectly conducting with a current distribution of $\boldsymbol{J}(x)e^{-jk_z z}$. The region $y > T$ is free space, with parameters (μ_0, ϵ_0) and wave number $k_0 = \omega\sqrt{\mu_0\epsilon_0}$. In this region the fields may be expanded as the sum of TM_Y field and a TE_Y field using

$$\begin{pmatrix} A \\ F \end{pmatrix} = \frac{\hat{y}}{2\pi} \int_{-\infty}^{\infty} \begin{pmatrix} \tilde{A} \\ \tilde{F} \end{pmatrix} e^{-j(k_x x + k_y y + k_z z)} \, dk_x \tag{9}$$

where $k_y^2 = k_x^2 + k_z^2 - k_0^2$.

In the substrate, the fields are expanded in terms of right and left circular vector potentials. Individually, right or left circularly polarized fields can not satisfy the boundary condition of zero